Exclusion probability in parentage tests

Co-dominant genetic markers, such as DNA microsatellites, are used to resolve parentage disputes. Three examples consider uniparous offspring in different circumstances:

(1) one parent is wrongly identified,
(2) both parents are wrongly identified and
(3) where the genotype of one parent is not available.

In parentage testing, the usefulness of any co-dominant marker is defined as the probability of it making an exclusion. Each of the three familiar situations is expressed as a general formula. In addition, each general formula is transformed, i.e. expressed in powers of the allele frequencies ($p_i$) to help those who calculate practical examples.

1) **Given two parents and one offspring; exclude a parent.** An example of this is a familiar paternity case. The general equation is

$$P = \sum_{i=1}^{n} p_i (1-p_i)^2 \sum_{i>j=1}^{n} (p_i p_j) [4-3(p_i+p_j)]$$

which, expressed as powers of $p_i$, becomes

$$P = 1-2\sum_{i=1}^{n} p_i^2 + \sum_{i=1}^{n} 2p_i^3 - 3\sum_{i=1}^{n} p_i^4 - 2(\sum_{i=1}^{n} p_i^2)^2 + 3\sum_{i=1}^{n} p_i^3 \sum_{i=1}^{n} p_i$$

(Jamieson 1965, 1979)

2) **Given two parents and one offspring; exclude both parents.** An example of this is a changeling. The general equation is

$$P = 1+\sum_{i=1}^{n} p_i^2 (2-p_i)^2 - 2\sum_{i=1}^{n} (2-p_i)^2 + 4(\sum_{i=1}^{n} p_i^3)^2 - 4p_i^6$$

(Grundel & Reets 1981)

which, expressed in powers of $p_i$, becomes

$$P = 1+4\sum_{i=1}^{n} p_i^4 - 4\sum_{i=1}^{n} p_i^5 - 8(\sum_{i=1}^{n} p_i^2)^2 + 8(\sum_{i=1}^{n} p_i^3)(\sum_{i=1}^{n} p_i^2) + 2(\sum_{i=1}^{n} p_i^3)^2$$

(Jamieson & Taylor 1997)

3) **Given one parent and one offspring; exclude their relationship.** An example of this is a sire or dam genotype missing. The general equation is

$$P = \sum_{i=1}^{n} p_i^2 (1-p_i)^2 + 2p_i p_j (1-p_i-p_j)^2$$

(Grabar & Morrice 1983)

which, expressed in powers of $p_i$, becomes
\[ P = \prod_{i=1}^{n} \left( 1 - \frac{1}{4} \sum_{i=1}^{n} (S_{pi_i}^2 + 2 (S_{pi_i}^2)^2 + 4 (S_{pi_i}^3)^2 - 3 S_{pi_i}^4) \right) \]  
(Jamieson & Taylor 1997)

Combining \( P \) over \( k \) unlinked markers in any of the above formulae gives

\[ P = 1 - (1 - P_1)(1 - P_2)(1 - P_3) \ldots \ldots (1 - P_k) \]

Derivations of formulae


