Exclusion probability in parentage tests

Co-dominant genetic markers, such as DNA microsatellites, are used to resolve parentage disputes. Three examples consider uniparous offspring in different circumstances:

(1) one parent is wrongly identified,
(2) both parents are wrongly identified and
(3) where the genotype of one parent is not available.

In parentage testing, the usefulness of any co-dominant marker is defined as the probability of it making an exclusion. Each of the three familiar situations is expressed as a general formula. In addition, each general formula is transformed, i.e. expressed in powers of the allele frequencies (p_i) to help those who calculate practical examples.

1) Given two parents and one offspring; exclude a parent. An example of this is a familiar paternity case. The general equation is

$$P = \sum_{i=1}^{n} p_i(1-p_i)^2 - \sum_{i>j=1}^{n} (p_i p_j)^2[4-3(p_i+p_j)] \qquad \text{(Jamieson 1965, 1979)}$$

which, expressed as powers of p_i, becomes

$$P = 1-2\sum_{i=1}^{n} p_i^2+\sum_{i=1}^{n} p_i^3+2\sum_{i=1}^{n} p_i^4-3\sum_{i=1}^{n} p_i^5-2(\sum_{i=1}^{n} p_i^2)^2+3\sum_{i=1}^{n} p_i^2\sum_{i=1}^{n} p_i^3 \qquad \text{(Jamieson 1994)}$$

2) Given two parents and one offspring; exclude both parents. An example of this is a changeling. The general equation is

$$P = 1+\sum_{i=1}^{n}[p_i^2(2-p_i)]^2 -2[\sum_{i=1}^{n} p_i^2(2-p_i)]^2+4(\sum_{i=1}^{n} p_i^3)^2-4\sum_{i=1}^{n} p_i^6 \qquad \text{(Grundel \& Reets 1981)}$$

which, expressed in powers of p_i, becomes

$$P = 1+4\sum_{i=1}^{n} p_i^4-4\sum_{i=1}^{n} p_i^5-3\sum_{i=1}^{n} p_i^6-8(\sum_{i=1}^{n} p_i^2)^2+8(\sum_{i=1}^{n} p_i^2)(\sum_{i=1}^{n} p_i^3)+2(\sum_{i=1}^{n} p_i^3)^2 \qquad \text{(Jamieson \& Taylor 1997)}$$

3) Given one parent and one offspring; exclude their relationship. An example of this is a sire or dam genotype missing. The general equation is

$$P = \sum_{i=j}^{n} p_i^2(1-p_i)^2+\sum_{i>j=1}^{n} 2p_i p_j(1-p_i-p_j)^2 \qquad \text{(Grabar \& Morrice 1983)}$$

which, expressed in powers of p_i, becomes

$$P = 1-4\sum_{i=1}^{n} p_i^2+2\left(\sum_{i=1}^{n} p_i^2\right)^2+4\sum_{i=1}^{n} p_i^3-3\sum_{i=1}^{n} p_i^4$$ (Jamieson & Taylor 1997)

Combining P over k unlinked markers in any of the above formulae gives

$$P = 1-(1-P_1)(1-P_2)(1-P_3)\ldots\ldots\ldots(1-P_k)$$

Derivations of formulae

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